Artificial Intelligence

Lecture 14

Symbolic Logic

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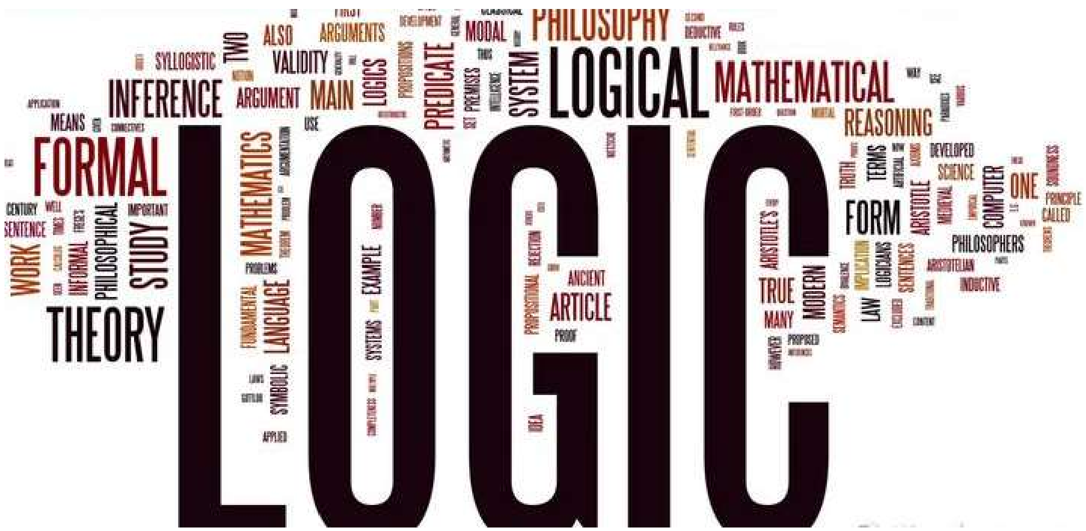
Symbolic Logic

Lecture Outlines:

- **Logic and Symbolic Logic**
- **Logic Types**
- **Propositional Logic**
  - Syntax
  - Semantic
  - Terminology
- …



Logic

- *Logic is concerned with the truth of statements about the world.* Generally each statement is either *TRUE* or *FALSE*.

- **Logic includes : *Syntax , Semantics and Inference Procedure*.**
  - **Syntax :** Specifies the *symbols* in the language about how they can be combined to form sentences. The facts about the world are represented as sentences in logic.
  - **Semantic :** Specifies how to assign a truth value to a sentence based on its *meaning* in the world. It Specifies what facts a sentence refers to. A fact is a claim about the world, and it may be *TRUE* or *FALSE*.
  - **Inference Procedure :** Specifies *methods* for computing new sentences from an existing sentences.

Facts are claims about the world that are *True* or *False*. ***Representation*** is an expression (sentence), stands for the objects and relations. ***Sentence*s** can be encoded in a computer program.

Symbolic Logic

- **Symbolic logic, The language of modern logic,** is a way to represent logical expressions by using symbols and variables in place of natural language, such as English, in order to remove vagueness.
  - **Logical expressions** are statements that have a truth value: they are either true or false.
  - A question like 'Where are you going?' or a command such as 'Stop!' has **no truth value**.
  - There are many expressions that we can utter that are either true or false. For example: All glasses of water contain 0.2% alcohol. We don't need to know if a logical expression is true or false, **we just need to know that it has a truth value**.

"If, if the first then the second and if the second then the third, then, if the first then the third."

$$[(p \supset q) \bullet (q \supset r)] \supset (p \supset r)$$

- It is the simplest kind of logic. Its modern development began with George Boole in the 19th century.

Logic as a KR Language

- *Logic is the Art of correct Reasoning*. In AI, logic is a language for reasoning, a collection of rules used while doing logical reasoning.
- Logic is a formal system in which the formulas or sentences have true or false values.

- The problem of designing a KR language is a tradeoff between that which is :

  (a) *Expressive* enough to represent important objects and relations in a problem domain.

  (b) *Efficient* enough in reasoning and answering questions about implicit information in a reasonable amount of time.

Logic Types…

- Logics are of different types : *Propositional logic, Predicate logic, Temporal logic, Modal logic, Description logic* etc.
- Two main subfileds of symbolic logic: **Propositional logic and Predicate logic**. These are fundamental to all logic.

    - **Propositional Logic (Propositional calculus)** is the study of the properties of propositions or statements and their connectivity. The propositions formed from "constants" and logical "operators".
    - **Predicate Logic (Predicate calculus)** expands on propositional logic by introducing variables, usually denoted by x, y, z, or other lowercase letters. It also introduces sentences containing variables, called predicates, usually denoted by an uppercase letter followed by a list of variables, such as P(x) or Q(y,z).

Logic Types

- The term ***Temporal Logic*** has been broadly used to cover all approaches to reasoning about time and temporal information. (for example, "I am *always* hungry", "I will *eventually* be hungry", or "I will be hungry *until* I eat something"). It is sometimes also used to refer to **tense logic.**

- **Modal logic** is a type of formal logic that extends classical propositional and predicate logic to include operators expressing modality. A modal—a word that expresses a modality (i.e., qualifies a statement). For example, the statement "John is happy" might be qualified by saying that John is *usually* happy, in which case the term "usually" is functioning as a modal. The traditional modalities of truth, include possibility ("Possibly, *p*", "It is possible that *p*"), necessity ("Necessarily, *p*", "It is necessary that *p*"), and impossibility ("Impossibly, *p*", "It is impossible that *p*").

- **Description logics** (**DL**) are a family of formal knowledge representation languages. Many DLs are more expressive than propositional logic but less expressive than first-order logic. DLs are used in AI to describe and reason about the relevant concepts of an application domain. A description logic (DL) models *concepts*, *roles* and *individuals*, and their relationships.

Propositional Logic

- *Propositional logic* is fundamental to all logic. *Propositional logic is also called Propositional calculus, Sentential calculus, or Boolean algebra.*
  - *Propositional logic* tells the ways of joining and/or modifying entire propositions, statements or sentences to form more complicated propositions, statements or sentences, as well as the logical relationships and properties that are derived from the methods of combining or altering statements.
  - *A proposition is a statement,* which is a declarative sentence that can be either *TRUE* or *FALSE,* but not both.
  - Examples: (a) The sky is blue., (b) Snow is cold. , (c) 12 * 12=144
  - A sentence is smallest unit in propositional logic.
    - if proposition is true, then truth value is "true" .
    - if proposition is false, then truth value is "false" .

PL: Syntax and Semantics

- **Syntax:**
  - Valid statements in PL are determined according to the rules of PL syntax.
  - The syntax governs the combination of basic building blocks such as propositions and logical connectives.
  - The syntax of PL is defined recursively as follows:
    - T and F are formulas.
    - If P and Q are formulas, the following are formulas:
      - (P)
      - (P & Q)
      - (P V Q)
      - $(P \rightarrow Q)$
      - $(P \Leftrightarrow Q)$
  - An example of a compound formula: $((P \& (Q \vee R) \rightarrow (Q \Leftrightarrow S))$
- **Semantics:**
  - The meaning of a sentence is just the value **true or false**.

PL: Terminology…

- These and few more related terms, such as, ***statements*, *connective*, *truth value*, *contingencies*, *tautologies*, *contradictions*, *antecedent*, *consequent and argument***.

- **Statement:**
  - *Simple statements* (sentences), *TRUE* or *FALSE*, that does not contain any other statement as a part, are *basic propositions*; lower-case letters, *p, q, r,* are symbols for simple statements.
  - *Large, compound or complex statement* are constructed from basic propositions by combining them with *connectives*.

PL: Terminology…

- **Connectives or Operators:**
  - The *connective*s join simple statements into compounds, and joins compounds into larger compounds.
  - Example of a formula : $((((a \land \neg b) \lor c \rightarrow d) \leftrightarrow \neg (a \lor c))$
  - Five basic connectives and their symbols:

Connectives and Symbols in decreasing order of operation priority

Connective	Symbols					Read as
assertion	P					"p is true"
negation	¬p	~	!		NOT	"p is false"
conjunction	p ∧ q	·	&&	&	AND	"both p and q are true"
disjunction	P v q	||	|		OR	"either p is true, or q is true, or both "
implication	p → q	⊃	⇒		if ..then	"if p is true, then q is true" " p implies q "
equivalence	↔	≡	⇔		if and only if	"p and q are either both true or both false"

PL: Terminology…

- **Truth value:**
  - The truth value of a statement is its *TRUTH* or *FALSITY.*

    Example :

    p is either *TRUE* or *FALSE*,

    ~p is either *TRUE* or *FALSE*,

    p v q is either *TRUE* or *FALSE*, and so on.

    use " **T** " or " **1** " to mean *TRUE*.

    use " **F** " or " **0** " to mean *FALSE*

- ***Truth table*** defining the basic *connectives* :

p	q	¬p	¬q	p ∧ q	p v q	p→q	p ↔ q	q→p
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	F	T	T	F	F
F	F	T	T	F	F	T	T	T

PL: Terminology…

- **Tautologies:**
  - A proposition that is always true is called a *tautology*. e.g., ***(P*** v ¬***P)*** is always true regardless of the truth value of the proposition ***P***.
- **Contradictions:**
  - A proposition that is always false is called a *contradiction.* e.g., ***(P*** ∧ ¬***P)*** is always false regardless of the truth value of the proposition ***P***.
- **Contingencies:**
  - A proposition is called a *contingency,* if that proposition is neither a *tautology* nor a *contradiction* e.g., ***(P*** v ***Q)*** is a contingency.

PL: Terminology…

- **Antecedent, Consequent:**
  - In the conditional statements, $p \rightarrow q$; the 1st statement or "if - clause" (here p) is called *antecedent*, 2nd statement or "then - clause" (here q) is called *consequent*.
- **Argument:**
  - Any argument can be expressed as a compound statement.
  - Take all the premises, conjoin them, and make that conjunction the antecedent of a conditional and make the conclusion the consequent. This implication statement is called the corresponding conditional of the *argument.*

Symbolic Logic

TO BE CONTINUED…