Artificial Intelligence

Lecture 24

Uncertainty

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Outlines

- …
- Methods to handle uncertainty
    - Fuzzy Logic
    - Bayesian Probability Theory
- …

Fuzzy Logic

- Traditional logic and sets are thought of as **crisp**
  - An item is T or F, an item is in the set or not in the set

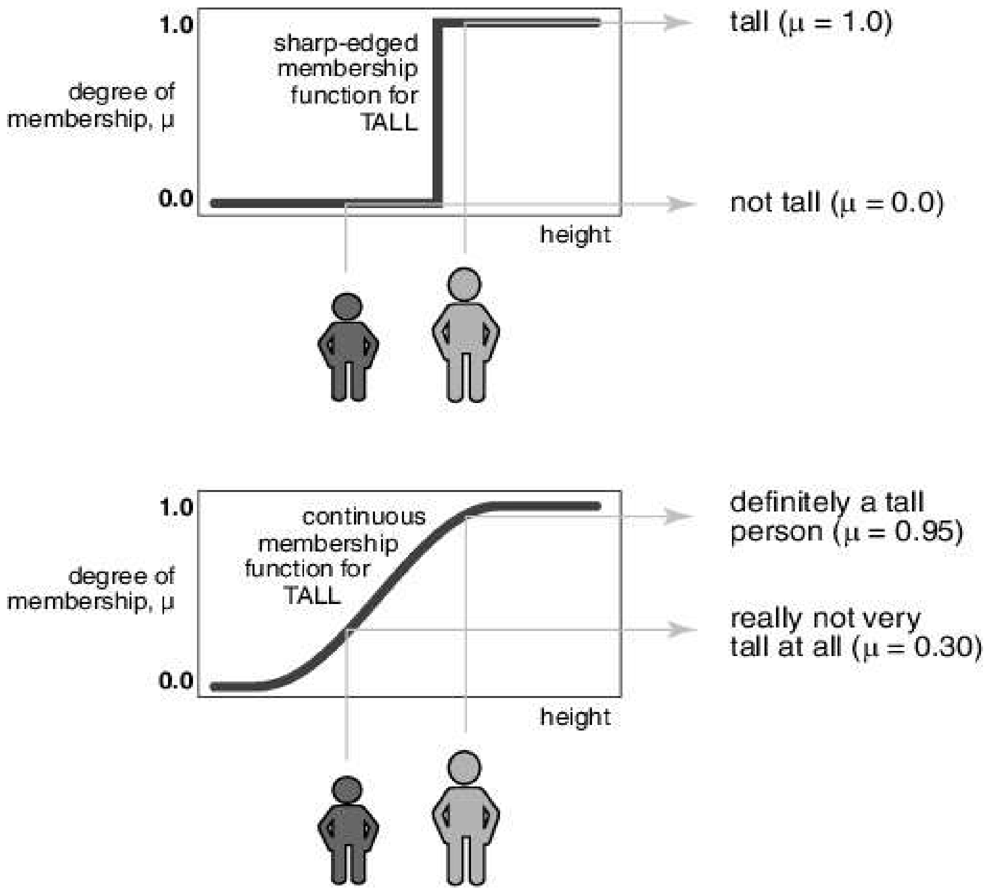
- Fuzzy logic, based on fuzzy set theory, says that an item is in a set by **f(a) amount**, known as membership value.
  - where a is the item
  - and f is the membership function (which returns a real number from 0 to 1)

- Membership to set A is often written like this:

$$A = \{x / \mu_A(x) \mid x \in X\}$$

Fuzzy Logic

- Consider the following figure that compares the crisp and fuzzy membership functions for “Tall”:



How to define fuzzy sets?

- A crisp set $C \subseteq S$ is defined by a characteristic function, $\chi_C(s)$: $S \rightarrow \{0, 1\}$.

$$X_C(s) = \begin{cases} 0 & if\ s \notin C \\ 1 & if\ s \in C \end{cases}$$

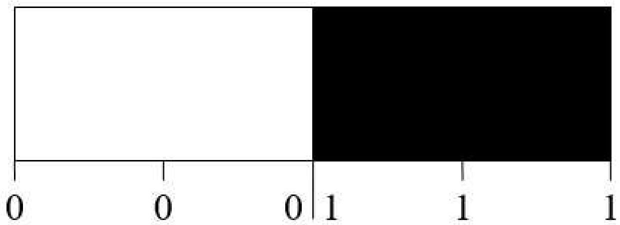
- A fuzzy set $F \subseteq S$ is defined by a membership function, $\mu_F(s)$: $S \rightarrow [0.0, 1.0]$.

$$\mu_F(s) = \begin{cases} 0.0 & if\ s\ is\ not\ in\ F \\ 0.0 < m < 1.0 & if\ s\ is\ partially\ in\ F \\ 1.0 & if\ s\ is\ totally\ in\ F \end{cases}$$

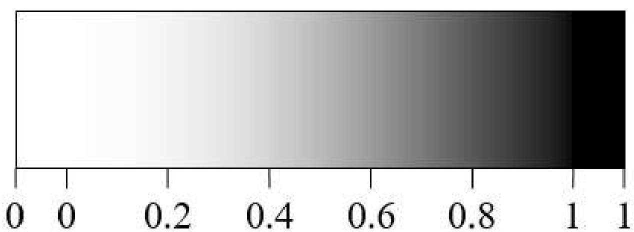
  - $\mu_F(s)$ describes to what *degree s* belongs to F: 1.0 means "definitely belongs", 0.0 means "definitely does not belong", other values indicate intermediate "degrees" of belonging.

How to define fuzzy sets?

- Range of logical values in Boolean and fuzzy logic:



(a) Boolean Logic.



(b) Multi-valued Logic.

- Consider N, the set of positive integers. Let $F \subset N$ be the set of "small integers”.

Let μ_F be like this:

$\mu_F(1) = 1.0$

$\mu_F(2) = 1.0$

$\mu_F(3) = 0.9$

$\mu_F(4) = 0.8$

...

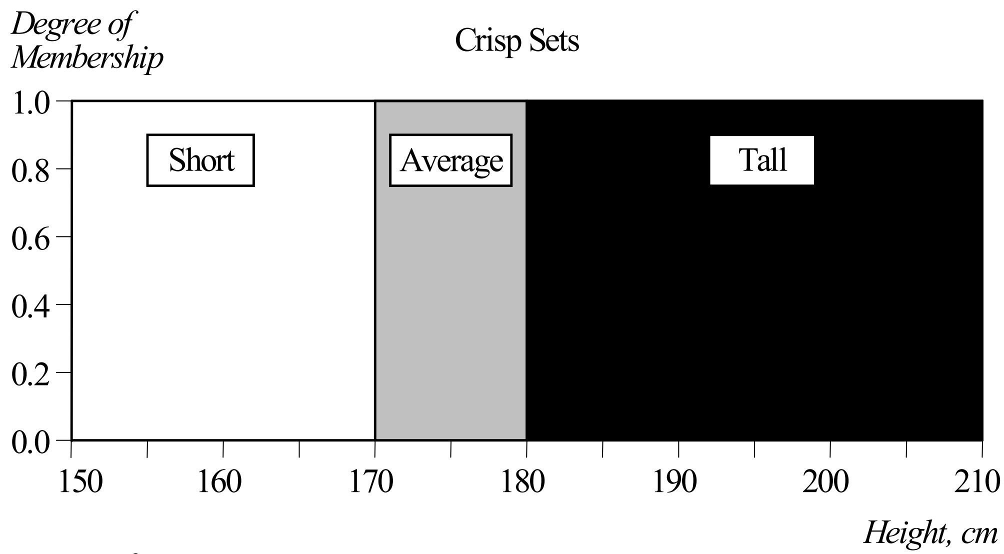
$\mu_F(50) = 0.001$

...

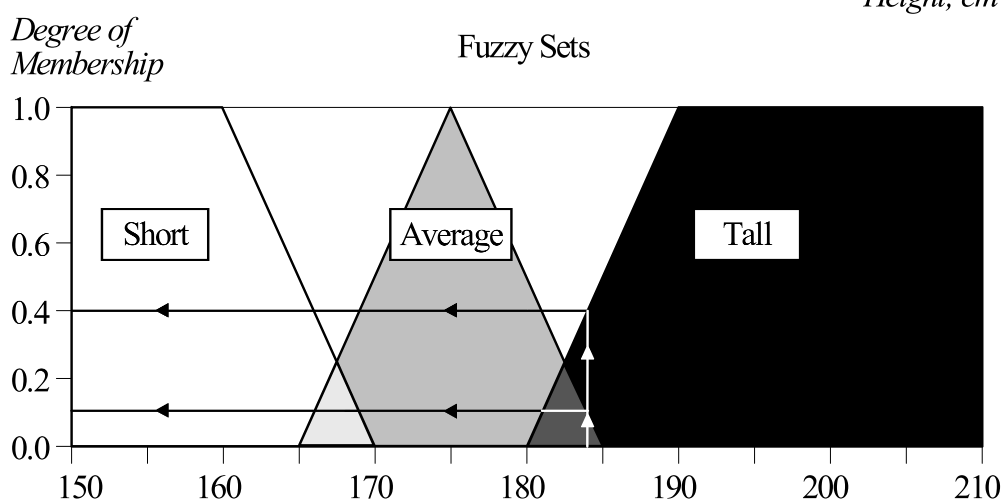
- μ_F defines a probability distribution for statements such as "X is a small integer".

How to define fuzzy sets?

- Sets of short, average and tall men



- .. and a man 184 cm tall



Bayesian Probability Theory

- **Bayes' Theorem**
- In probability theory and statistics, Bayes' theorem (Bayes's law) describes the probability of an event, based on prior knowledge of conditions that might be related to the event.
  - Medical diagnosis is a handy example of Bayes' Theorem. If the risk of developing health problems is known to increase with age, Bayes' theorem allows the risk to an individual of a known age to be assessed more accurately than simply assuming that the individual is typical of the population as a whole.
  - A patient may have a cold, a flu, pneumonia, rheumatism, and so on. The usual symptoms are high fever, short breath, runny nose, and so on.
  - We need the probabilities (based on statistical data) of all diseases, and the probabilities of high fever, short breath, runny nose in the case of a cold, a flu, pneumonia, rheumatism.
  - We would also like to assume that all relationships between H_j and E are mutually independent.

Bayesian Probability Theory

- **Bayes' Theorem**
- Bayes' theorem allows us to compute how probable it is that a hypothesis H_i follows from a piece of evidence E (for example, from a symptom or a measurement).
  - The idea is that you are given some evidence $E = \{e_1, e_2, \ldots, e_n\}$ and you have a collection of hypotheses $H_1, H_2, \ldots, H_m$
  - The required probability data: $p(H_i|E)$ the probability of H_j, $p(H_i)$ the overall probability of H_i and $p(E|H_i)$ the probability of E given H_j for all possible hypotheses.

  - **Bayes' theorem:**
  - $p(H_i|E) = \frac{p(E|H_i) * p(H_i)}{\sum_j p\left(E\middle|H_j\right) * p(H_j)}$

Bayesian Probability Theory

- **Bayes' Theorem :**

$$p(H_i|E) = \frac{p(E|H_i) * p(H_i)}{\sum_j p(E|H_j) * p(H_j)}$$

  – If we assume that all the conditional probabilities under summation are independent, we can simplify the formula:

$$p(H_i|E) = \frac{p(E|H_i) * p(H_i)}{p(E)}$$

Uncertainty

TO BE CONTINUED…